

# **Practical guide to financial and insurance mathematics**

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# 1. INTRODUCTION

This publication presents basic calculation principles in finance and insurance. Particular chapters gradually survey topics which are important in the actual financial and insurance practice. Theoretical description of financial and insurance methods and instruments including corresponding formulas are demonstrated by numerical examples which are typical for the given subjects and highlight actual applications (e.g. mortgages, credit life insurance and others). Moreover, the examples are mostly numerically simple so that they can be realized using calculators with mathematical functions without applying any financial software (it predetermines the text as a suitable training material in contrast to computer oriented courses of finance).

Roughly speaking, Chapters 2–12 are devoted to finance and Chapters 13–17 to insurance. At first basic interest and discount concepts are described and generalized in Chapters 2–4. Systems of cash flows including investment rules, financial annuities and debt amortization are presented in Chapters 5–7. Several chapters are devoted to securities, namely the short-term securities (Chapter 3), the bonds (Chapter 8), the stocks (Chapter 9), the derivative securities (Chapter 10) and the speculation on securities (Chapter 11). Financial risk and portfolio analysis are studied in Chapter 12. The remaining chapters deal with insurance topics, namely the life insurance including the life tables (Chapters 14–15), the pension insurance (Chapter 16) and the non-life insurance (Chapter 17). In addition to solved examples, each chapter contains a section of unsolved examples which enable to exercise the material of each chapter checking one's results with the ones given in the appendix. The second appendix presents financial tables that can be useful for a quick evaluation of practical financial problems (even though such tables may look as an anachronism in the computer era).

Primarily the publication is proposed for university students of various English programs of quantitative economic analysis (QEA) or, particularly, programs of finance and actuarial sciences (or possibly for Czech students preferring “more international” approach to the finance and insurance with English terminology). The publication may be helpful also for people dealing professionally with problems of quantitative economy (in banks, insurance companies and pension funds, financial institutions, investment planning, treasury, government bodies and others) or simply are interested in financial or insurance products they meet in practical life.

The text uses standard symbols of financial and actuarial mathematics. References to examples, remarks and formulas are organized in a natural way, e.g. the Example 6.2.1 is the first example in Section 6.2 of Chapter 6. The ending of examples and remarks is always highlighted by symbol  $\square$ .

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